

# Spectroscopy through the change of undulator parameters in dipole regime

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## Abstract

In this work a method of spectroscopy without monochromators for the undulator radiation (UR) source is proposed. This method is based of changing the magnetic field in the undulator. Different variations of field modulations and corresponding object reaction for the case of a dipole regime of UR exitation were considered. The results of a numerical experiment are shown and possibilities of this method for an undulator consisting of two blocks and rearranging by changing the distance between the blocks were estimated.

## 1. Introduction

In recent works [1-3] it has been proposed to use undulator radiation for spectroscopy without monochromators. According to these proposals the UR without an intermediate converter falls on the spectroscopical object, where the spectral density of the UR flux is multiplied with the spectral sensitivity of the detector on every wavelength  $\lambda$  and then summed. This summation gives an object reaction on the UR flux integrally over all wavelengths. Considering the UR spectrum can be relativistically changed (by changing the particles energy or the undulator field), the reaction of the object on the rearranged source of radiation can be defined as an integral Fredholm equation of the  $I$  kind:

$$I(P) = \int_0^{\infty} K(P, \lambda) A(\lambda) d\lambda, \quad (1)$$

where  $I(P)$  is the object reaction on the UR flux,  $K(P, \lambda)$  is the equation kernel (1) - spectral density of the radiation flux,  $A(\lambda)$  is the spectral function of the radiation acceptor, and  $P$  is the rearranging parameter. The determination of the object function  $A(\lambda)$  by the measured reaction  $I(P)$  reduces to the solution of the integral equation with the known kernel  $K(P, \lambda)$ .

As was shown in Ref. [2], the simplest way to solve this equation is to use the energy of the accelerated particles as a rearranging parameter. This method was

already used in Refs. [2,3]. However, the active change of the particle energy is effectively possible only for accelerators and rather problematic by big storage rings, to which the majority of users are now oriented. Spectroscopy without monochromators attract interest, because in this case the spectral properties of the source could be changed not by the particle energy, but by changing the undulator field.

## 2. Basic relations

Singe spectroscopy without monochromators produces no loss in intensity of the source on an intermediate converter, it is suitable to use an undulator with the dipole regime of UR exitation for the dipole parameter  $k^2 \ll 1$ . By this means, spectroscopy without monochromators is confined to changes in the UR field structure (not taking into consideration changes of  $k^2$ ), but it gives, nowever, a simler possibility for the numerical realization of the algorithm of the solution of Eq. (1). The spectral density of the photon flux in an arbirary spectral interval  $d\bar{\omega} = d\omega/\omega$  for a radiating charge in the dipole approximation, according to Refs[2,3], can by written as:

$$\frac{d\Phi}{d\bar{\omega}} = \frac{8\pi\alpha}{e} \left( \frac{\mu_0 e}{\pi m c} \right)^2 J \eta \int_{\eta}^{\infty} H^2(\nu) \left( 1 - 2\frac{\eta}{\nu} + 2\frac{\eta^2}{\nu^2} \right) \frac{d\nu}{\nu^2}, \quad (2)$$

where  $J$  is the magnitude of the current of the accelerated charge in the storage ring,  $\alpha$  is the fine structure constant,  $e$  is the electron charge,  $m$  is the electron mass,  $\mu_0$  is the magnetic permeability of vacuum,  $\eta = (\lambda_0/(2\lambda\gamma^2))(1 + k^2)$  is the number of the UR harmonic;  $\gamma$  is the Lorentz factor,  $\lambda_0$  is the period length of the undulator magnetic field, and  $H^2(\nu)$  is the square modulus of the Fourier-structure of the UR magnetic field.

As was shown in Ref. [3] for spectroscopy without monochromators an undulator consisting of periodical blocks is suitable. Its Fourier structure of the field can be written as follows:

$$H(\nu) = G(\nu)\Psi(\nu)S(\nu), \quad (3)$$

where  $G(\nu)$  is the Fourier structure of a standart undulator element (UE) (i.e. an element from which an undulator half-period is formed),  $\Psi(\nu)$  is the Fourier structure of an undulator block, as a set of UE,  $S(\nu)$  is the Fourier structure of the undulator, as a set blocks.  $G(\nu)$  is the expression for an ironless electro-magnetic undulator with the winding of an optimal profile section [3]:

$$G(\nu) = \frac{j\lambda_0^2}{2} \frac{e^{-2\pi\nu h/\lambda_0}}{(\pi\nu)^2} \left( \cos\left(\frac{\pi\nu}{4}\right) - \cos\left(\frac{\pi\nu}{2}\right) - \frac{\pi\nu}{4} \sin\left(\frac{\pi\nu}{4}\right) \right), \quad (4)$$

where  $j$  is the current density in winding section UE and  $2h$  is the magnetic gap. For  $n$ -fold balanced charge motion in the undulator block (this corresponds to

the condition of turning to zero of the  $n$ -fold integral from the UR field along undulator block), with period number  $N$ , the Fourier structure is [3]:

$$\Psi(\nu) = \left( 2 \cos \left( \frac{\pi}{2} (1 + \nu) \right) \right)^n \cdot \frac{\sin \left( \frac{\pi}{2} (1 + \nu) (2N - n) \right)}{\sin \left( \frac{\pi}{2} (1 + \nu) \right)}. \quad (5)$$

If all  $M$  undulator blocks are equal and the distances between neighbouring blocks are  $l$ , then for  $S(\nu)$  one can derive from Ref. [3]:

$$S(\nu) = 2 \cos \left( \frac{\pi}{2} (\delta + \nu(2N + L)) \right) \cdot \frac{\sin \left( \frac{\pi}{2} (\delta + \nu(2N + L)) M \right)}{\sin \left( \frac{\pi}{2} (\delta + \nu(2N + L)) \right)}, \quad (6)$$

where  $L = 2l/\lambda_0$ , i.e. it defines the distances between the blocks in units of half-periods, and parameter  $\delta$  defines the phasing of the blocks switching on: the value  $\delta = 0$  corresponds to the in phase switching on; the value  $\delta = 1$  corresponds to switching on in the opposite phase.

As was shown in Ref. [3] for the purpose of monochromator-free spectroscopy the integral spectrum, taken as the difference in the integral spectra corresponding to the undulator state with in phase and opposite phase switching on, is most suitable as the kernel of Eq. (1) (further, the differential UR kernel). Since for such a kernel the low frequency range of the spectrum is suppressed and accordingly the contributions of "off-axis" particles into the radiation are also suppressed, the  $\sigma$ -component polarization appears. Further, by  $S(\nu)$  the expression corresponding to the phase state difference is meant.

### 3. Object reaction on UR flux

Undulators for obtaining arbitrary UR are described in detail in the literature. It is possible to modulate the properties of the UR flux for a multiblock undulator, as described above, either by changing the geometrical parameters of the UE, or by changing the contribution to undulator of the separate elements, or by changing the blocks location.

If the field is modulated by changing the geometrical parameters of the UE, then the most technically realized, in this case, is changing of the magnetic gap. However, if we consider the dipole excitation UR regime, then the corresponding changing interval  $h$  does not essentially change the form of the spectrum. This is clearly seen if one considers the integral UR spectrum according to Eq. (2) in the first UR harmonic approximation. In this case (2) becomes

$$\frac{d\Phi}{d\omega} = CG^2(h)\eta \int_{\eta}^{\infty} \Psi^2(\nu) S^2(\nu) \left( 1 - 2\frac{\eta}{\nu} + 2\frac{\eta^2}{\nu^2} \right) \frac{d\nu}{\nu^2} \quad (7)$$

( $C = \text{const.}$ ) and integral operator  $K(h, \eta(\lambda)) = d\Phi/d\omega$  is degenerate because it can be written as  $K(h, \eta) = K_1(h)K_2(\eta)$ . If we consider a set of harmonics, for

example the first and the third, then changing of  $h$  leads to a rearrangement of the contributions of these harmonics in the general spectrum.

Changing the contributions of the UE in the frame of expression (5), is really possible only for an electromagnetic undulator system, and even this is possible only in discrete limits. Therefore in this work this case is not considered.

So, the most simple variant of UR field modulation is modulation by changing the undulator blocks location, i.e., by changing the distance  $L$  between the blocks. It is obvious that an arbitrary  $L$  can be realized for  $L > 0$  (by mechanical removal), because for  $L < 0$  only discrete values exist (by rearranging the contributions into the undulator).

The reaction of an object with spectral sensitivity  $A(\lambda)$  by the full capture of the UR flux over all wavelengths changeable over the undulator  $L$  is defined as follows:

$$I(L) = \int_{\eta}^{\infty} A(\lambda) \frac{d\Phi}{d\omega}(\eta(\lambda), L) \frac{d\lambda}{\lambda} = \int_{\eta}^{\infty} A(\eta) \frac{d\Phi}{d\omega}(\eta, L) \frac{d\eta}{\eta} = \int_{\eta}^{\infty} A(\eta) K(\eta, L) d\eta. \quad (8)$$

At  $L \rightarrow \infty$ ,  $I(L)$  goes asymptotically to 0; that is why changing the interval of  $L$  is defined by giving the reaction level  $I(L)$ . On the other side, real limitations on changing of  $L$  exist. They are correlated with the value of the intervening space in the insertion device. Since the limits of changing of  $L$  define the method resolution ( $\Delta\lambda/\lambda \approx 1/L_{max}$ ), at a certain value of the intervening space  $L_{max}$ , it is better to use blocks with minimal values  $N$ , which guarantee a meaningful level  $I(L)$  over the whole interval, and for making the radiation characteristics better one has to use the maximal balancing degree of the block  $n = N - 1$ . A solution of Eq. (8)  $A(\eta)$  for the measured reaction  $I(L)$  and the analytically given equation kernel  $K(\eta, L)$  can be found by means of the Tichonov regularization method [4].

#### 4. Numerical model of the algorithm

Since the resolution  $\Delta\lambda/\lambda$  of the method of defining the object spectral function by its reaction (8) is a value of the order of  $1/L_{max}$  (where  $L_{max} \sim N$ ) and the number of calculation operations grows proportionally to product  $NL_{max}$ , then for a simple realization of a numerical model an undulator consisting of two blocks was used; there were four elements in every block with a balancing degree of every block  $n = 3$ . The corresponding contribution structure of the excitation currents of the blocks elements is written as [3]  $\{-1; 3; -3; 1\}$ .

As a function of spectral sensitivity of object  $A(\eta)$  a set of Gaussian was used:

$$A(\eta) = \sum_{i=1}^2 e^{(\eta-\eta_i)^2/\sigma_i^2}, \quad (9)$$

with parameters  $\eta_i = \{0.6; 0.9\}$ ,  $\sigma_i = \{0.3; 0.1\}$ , so that the reflection  $\eta =$

$\lambda_0/(2\lambda\gamma^2)$  gives the function:

$$A(\lambda) = \sum_{i=1}^2 e^{-(1-\lambda_i/\lambda)^2/\bar{\sigma}_i^2}, \quad \bar{\sigma}_i = \sigma_i/\eta_i, \quad (10)$$

corresponding to two spectral lines with the relation  $\Delta\lambda/\lambda$  with is equal to 0.35 and 0.08 respectively. (Relatively small values of  $\Delta\lambda/\lambda$  correspond to a choice of the undulator parameters made above).

The kernel of the equation was defined over the interval  $\eta \in (0, 2)$ , and the limits of changing of  $L$  were taken from 0 to 20. The size of the net, which approximates the kernel of Eq. (8) was taken equal to  $(200 \times 400)$ .

To solve Eq. (8) the Tichonov regularization method with a choice of the regularization parameter according to a generalization of nonbuilding principle [4] was used. For the minimization of Tichonov functional the method of gradients was used.

## 5. Conclusion

From the results of the numerical experiments one can see that the considered algorithm of monochromator-free spectroscopy may be rather perspective for insertion devices where an active changing of the particles energy is problematic. For a realization of this method a simple undulator, consisting of two blocks and rather small period number  $N$  in every block ( $\approx 5 - 20$ ) but with the opportunity to change the distances between the blocks, is necessary. The theoretical resolution of this method is inversely proportional to the distance between the blocks  $L_{max}$  (in units of half-periods). The object reaction practically disappears at a shift value  $L \approx 10N$ . This value, probably, defines the maximally possible method resolution by a given period number  $N$  in the block. So, if we consider a period number for each block  $N = 20$  and a period length of 2 cm, then we will obtain  $L_{max} \approx 200$  and accordingly the possible resolution,  $\Delta\lambda/\lambda \approx 1/L_{max} = 5 \cdot 10^{-3}$  (by the total length of such a twoblock system), is equal  $(2N + L_{max}) \cdot 2 = 480$  cm.

## References

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